Introduction to mean-field spin glasses *and the TAP approach*

David Belius

Faculty of Mathematics & Computer Science
FernUni Schweiz / UniDistance Suisse

FernUni.ch
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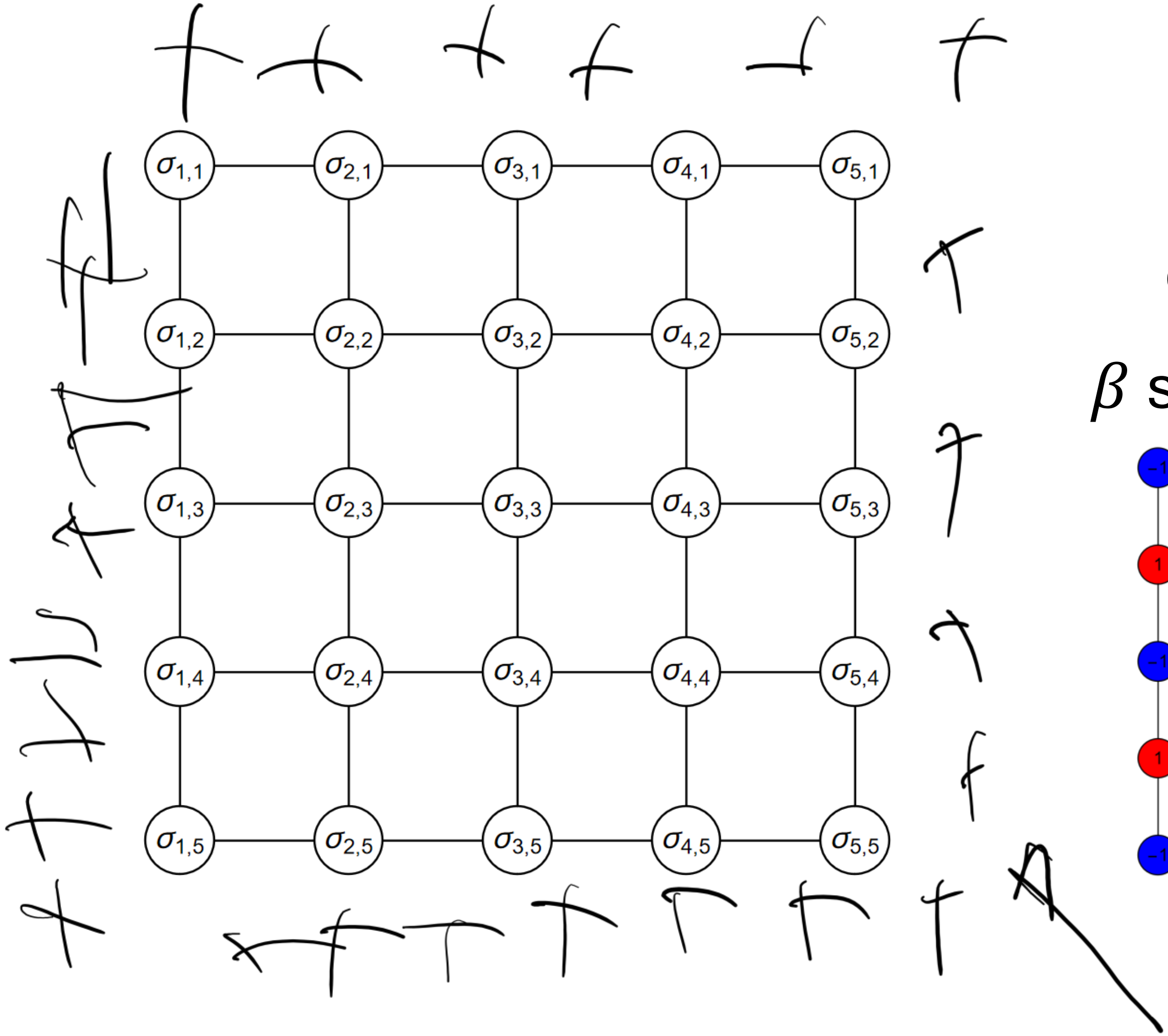
	Monday	Tuesday	Wednesday	Thursday	Friday
8.30-9.30	***Registration***				
09.30-10.20	BELIUS	BELIUS	SIMONELLA	SIMONELLA	DARIO
10.20-10.50	***coffee***	***coffee***	***coffee***	***coffee***	***coffee***
10.50-11.40	BISKUP	BERESTYCKI	PELED	BELIUS	BERESTYCKI
11.50-12.40	PELED	SIMONELLA	BERESTYCKI	BISKUP	BISKUP
12.40-15.00	***lunch***	***lunch***	***lunch***	***lunch***	***lunch***
15.00-15.50	BISKUP	BELIUS	***free afternoon***	SIMONELLA	PELED
15.50-16.20	***coffee***	***coffee***	***free afternoon***	***coffee***	***coffee***
16.20-17.10	short talks	short talks	***free afternoon***	short talks	BERESTYCKI
17.20-18.10	short talks	short talks	***free afternoon***	short talks	
18.30-20.00	***Wine & Cheese***				

EXERCISE CLASS

LECTURE

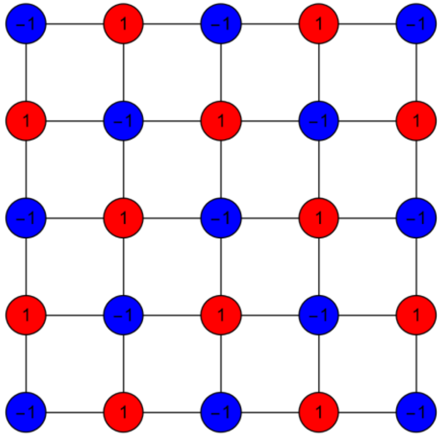
Spin model: Ising model (1920)



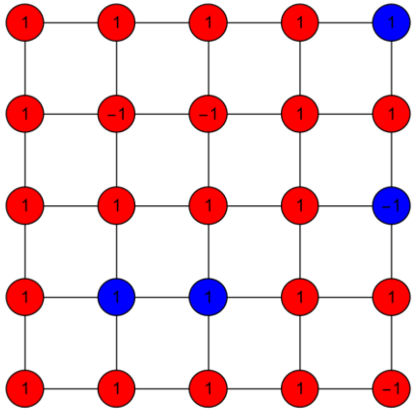
$\beta \in [0, \infty)$: β =strength of interaction= $\frac{1}{Temp}$

G_β probability measure on space of all spin configurations

β small: high temp



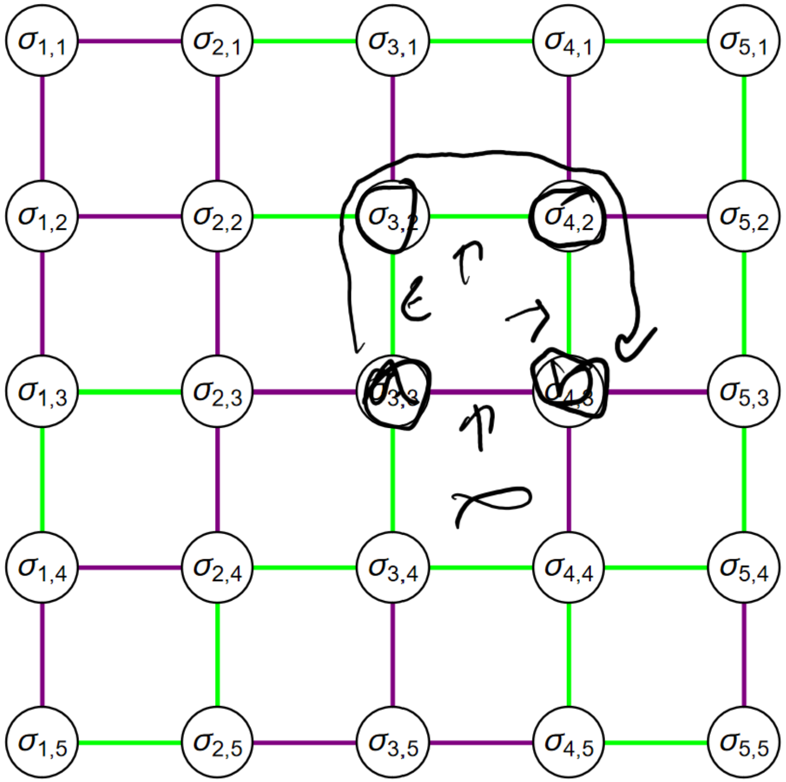
β large: low temp



Phase transition at $\beta = \beta_c$

$$G_\beta \approx \frac{1}{2} G_{\beta,-} + \frac{1}{2} G_{\beta,+}$$

Spin glass model: Edwards-Anderson (EA) model (1975)



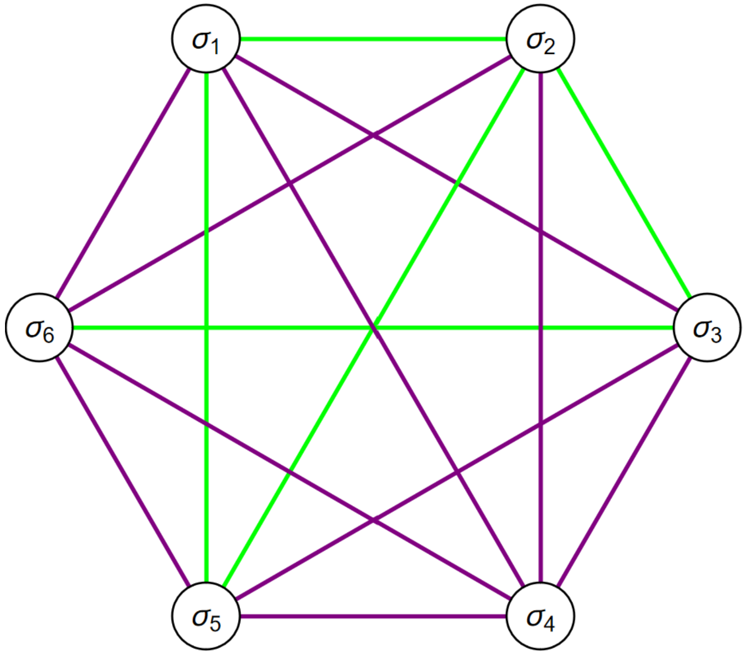
$\beta \in [0, \infty)$: β =strength of interaction= $\frac{1}{Temp}$

G_β probability measure on space of all spin configurations

Nature of phases?

Mean-field spin glass model: Sherrington-Kirkpatrick (SK) model (1975)

$N \to \infty$



$\sigma = (\sigma_1, \ldots, \sigma_N)$

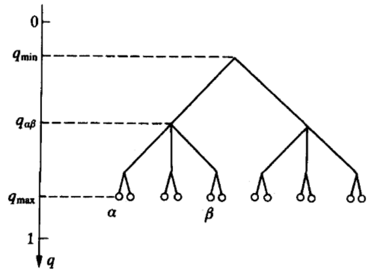
& $\sigma \in \mathbb{R}^N$

$\beta \in [0, \infty)$: β =strength of interaction= $\frac{1}{Temp}$

G_β probability measure on space of all spin configurations

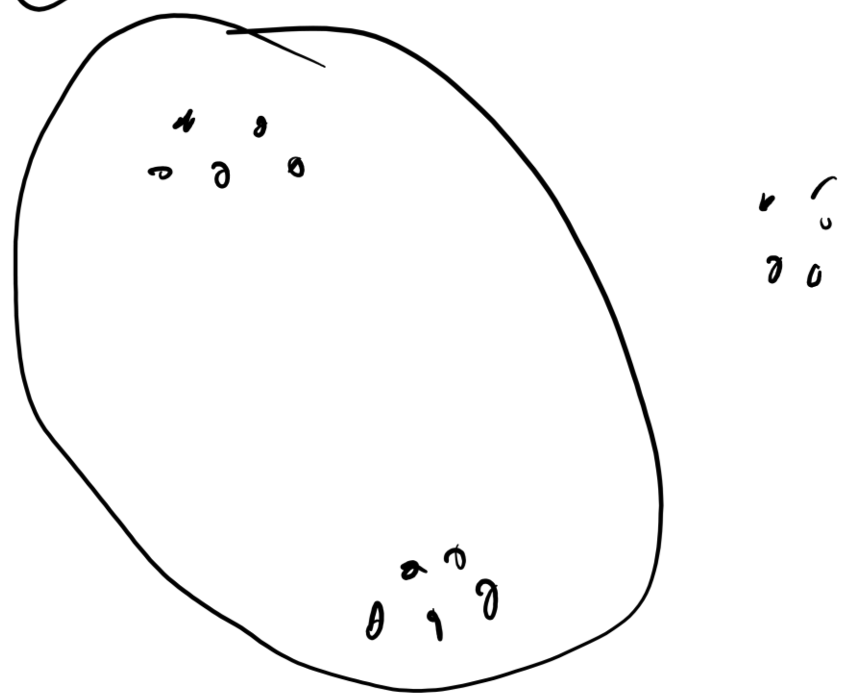
High temp: kind of like Ising, decorrelated spins

Low temp: $G_\beta = \sum u_m G_{\beta,m}$ (Parisi 1980)



Similar structure in hard combinatorial optimization problems

$$F(\sigma_1, \ldots, \sigma_N) = 0$$



Proper definitions

Curie-Weiss model (mean-field spin model)

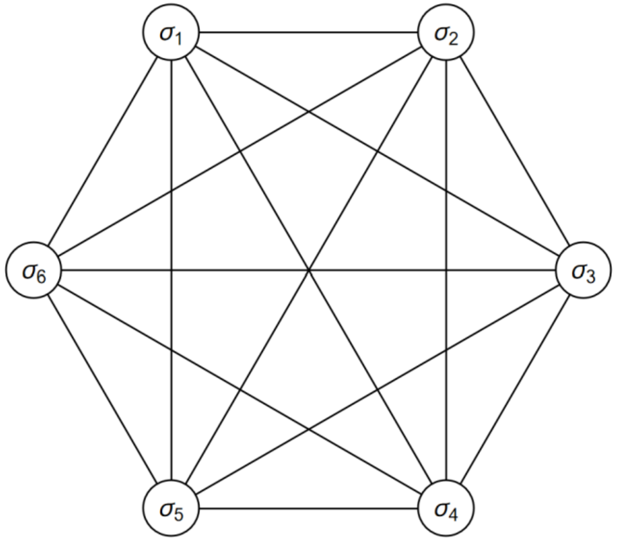
$(\sigma_1, \dots, \sigma_N)$

$$H_N(\sigma) = \sum_{i,j=1,\dots,N} \frac{1}{N}\sigma_i\sigma_j$$

$$H_N^h(\sigma) = H_N(\sigma) + h\sum_{i=1,\dots,N}\sigma_i$$

β, h: params

Q_N uniform on $\{-1,1\}^N$
- $Q_N[\sigma_i] = 0$
- σ i.i.d. under Q_N



Gibbs measure: $G_N(A) := \dfrac{Q_N\left[1_A \exp\left(\beta H_N^h(\sigma)\right)\right]}{Z_N}$

Partition function: $Z_N := Q_N\left[\exp\left(\beta H_N^h(\sigma)\right)\right]$

Free energy: $F_N := \dfrac{1}{N}\log Z_N$

Goal: compute $\lim F_N$

ISING SPHERICAL $S_{N-1}(\sqrt{N})$

Sherrington-Kirkpatrick (SK) model '75

SPHERE

$$\boxed{H_N(\sigma)} = \sum_{i,j=1,\dots,N} \frac{J_{ij}}{\sqrt{N}} \sigma_i \sigma_j$$

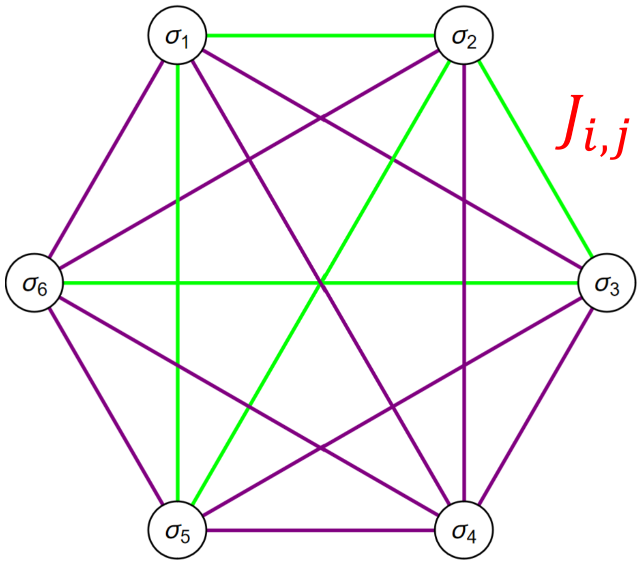
$$H_N^h(\sigma) = H_N(\sigma) + h \sum_{i=1,\dots,N} \sigma_i$$

β, h: params

UNIFORM ON

Q_N uniform on $\{-1,1\}^N$

- $Q_N[\sigma_i] = 0$
- σ i.i.d. under Q_N



Gibbs measure: $G_N(A) := \dfrac{Q_N\left[1_A \exp\left(\beta H_N^h(\sigma)\right)\right]}{Z_N}$

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Free energy: $F_N := \dfrac{1}{N} \log Z_N$ $\quad E[F_N]$

Goal: compute $\lim F_N$

THE END